International Journal of High-Rise Buildings
June 2021, Vol 10, No 2, 85-92
https://doi.org/10.21022/IJHRB.2021.10.2.85

International Journal of
High-Rise Buildings
www.ctbuh-korea.org/ijhrb/index.php

Practical Experience with Full-scale Performance Verification of Dynamic Vibration Absorbers installed in Tall Buildings

J.S. Love[1] and B. Morava[1]

[1]*Motioneering Inc., Canada*

Abstract

Dynamic vibration absorbers (DVAs) in the form of tuned sloshing dampers (TSDs) and tuned mass dampers (TMDs) are commonly used to reduce the wind-induced motion of high-rise buildings. Full-scale performance of structure-DVA systems must be evaluated during the DVA commissioning process using structural monitoring data. While the random decrement technique (RDT) is sometimes employed to evaluate the DVA performance, it is shown to have no theoretical justification for application to structure-DVA systems, and to produce erroneous results. Subsequently, several practical methods with a sound theoretical basis are presented and illustrated using simulated and real-world data. By monitoring the responses of the structure and DVA simultaneously, it is possible to directly measure the effective damping of the system or perform system identification from which the DVA performance can be evaluated.

Keywords: Dynamic vibration absorber (DVA), Tuned sloshing damper (TSD), Tuned mass damper (TMD), Wind-induced motion comfort, Structural control

1. Introduction

Modern high-rise buildings are typically slender, lightweight, flexible, and possess low inherent structural damping. As a result, they are susceptible to excessive wind-induced motions, which can lead to occupant discomfort, building "groaning", damage to non-structural elements, or elevator malfunction. While wind-induced motions can often be reduced by increasing the mass or stiffness of the structure, or by modifying the building shape, these options are expensive and often undesirable. Instead, design teams are frequently employing dynamic vibration absorbers (DVAs) in the form of tuned sloshing dampers (TSDs) and tuned mass dampers (TMDs) to decrease the resonant response of buildings.

A TMD is a mechanical device that is located near the top of a building, and is composed of a rigid mass that is connected to the structure through elements that provide a restoring force, such as springs or pendulum cables, and elements that dissipate energy, such as viscous damping devices. In practice, the simple pendulum TMD is most common, where the restoring force is provided by gravity. The natural frequency of the TMD is "tuned" to be close to the natural frequency of the building, and an appropriate amount of damping is provided (Warburton 1982). When the building experiences a resonant excitation, the TMD mass will respond by moving out-of-phase with the structure, thereby opposing its motion. Structure-TMD system monitoring is conducted by installing accelerometers on the structure and TMD mass to simultaneously measure the response of both degrees of freedom.

A TSD consists of a tank that is located near the top of a building and is partially filled with a liquid (typically water) that can slosh. By selecting the proper tank length, width, and quiescent water depth, the natural sloshing frequency of the tank can be "tuned" to the natural frequency of the structural mode of vibration to be controlled. Screens or baffles are typically installed to increase the TSD damping towards its optimal value (Tait 2008). In its simplest form, a TSD can be represented as an "equivalent TMD" with an equivalent mass, stiffness, and damping (Tait 2008). Structure-TSD system monitoring is conducted by installing accelerometers on the building and measuring the wave heights within the TSD tank using wave probes.

In this paper, practical methods for evaluating the performance of structure-DVA systems using structural monitoring are presented and illustrated with examples. First, the fundamentals of the response and performance of structure-DVA systems are described. Next, the random decrement technique (RDT) is discussed as a method that continues to be erroneously applied to structure-DVA systems. While RDT is a powerful tool for evaluating damping in structures without DVAs, it is shown to produce incorrect results when applied to structure-DVA systems. Lastly, several practical methods for evaluating the total effective damping of structure-DVA systems are presented and demonstrated using numerical simulations and real-world structural monitoring data.

†Corresponding author: Shayne Love
Tel: +1-519-823-1311
E-mail: Shayne.Love@motioneering.ca

2. Background

2.1. Structure-DVA Systems

The TMD and TSD can both be represented as auxiliary spring-mass-dashpot systems connected to a structure (see Fig. 1(a)). Although the damping from these systems is typically nonlinear, through statistical linearization the damping can be represented as amplitude-dependent equivalent viscous damping (Caughey 1963; Rüdinger 2007; Tait 2008). Herein, the generalized mass, natural frequency, and inherent damping ratio of the structure are M_s, ω_s, and ζ_s, respectively, and the equivalent mass, natural frequency, and damping ratio of the DVA are given by m_d, ω_d, and ζ_d, respectively. The mass ratio of the DVA and structure is defined as $\mu = m_d / M_s$. Increasing μ will increase the acceleration reduction performance provided by an optimally-tuned system (McNamara 1977; Warburton 1982). The mechanical admittance function for the structure and DVA are given by (McNamara 1977):

$$H_s(\omega) = \frac{-\omega^2 B_2 + i\omega B_1 + B_0}{\omega^4 A_4 - i\omega^3 A_3 - \omega^2 A_2 + i\omega A_1 + A_0} \tag{1}$$

$$H_d(\omega) = \frac{\omega^2 B_2}{\omega^4 A_4 - i\omega^3 A_3 - \omega^2 A_2 + i\omega A_1 + A_0} \tag{2}$$

where:

$$A_0 = \omega_s^2 \omega_d^2$$

$$A_1 = 2\zeta_s \omega_s \omega_d^2 + 2\zeta_d \omega_d \omega_s^2$$

$$A_2 = \omega_s^2 + \omega_d^2(1+\mu) + 4\zeta_s \zeta_d \omega_s \omega_d$$

$$A_3 = 2\zeta_s \omega_s + 2\zeta_d \omega_d (1+\mu) \quad A_4 = 1$$

$$B_0 = \omega_s^2 \omega_d^2 \quad B_1 = 2\zeta_d \omega_d \omega_s^2 \quad B_2 = \omega_s^2$$

Figure 1(b) shows typical mechanical admittance functions for a bare structure and a structure with a DVA. In this example, the structural properties are $\omega_s = 1.0$ rad/s (0.16 Hz), and $\zeta_s = 1\%$, and the DVA properties are $\mu = 1\%$, $\omega_d = 0.99$ rad/s, and $\zeta_d = 5.0\%$, which are the optimal values for this system. The inclusion of an optimally tuned DVA leads to a significant reduction of the peak resonant response, which is replaced with two much lower peaks. If the structure is subjected to white noise excitation of spectral magnitude, S_0, the response variance is computed as the square of the area under the curve shown in Figure 1(b), that is

$$\sigma_s^2 = S_0 \int_0^\infty |H_s(\omega)|^2 d\omega \tag{4}$$

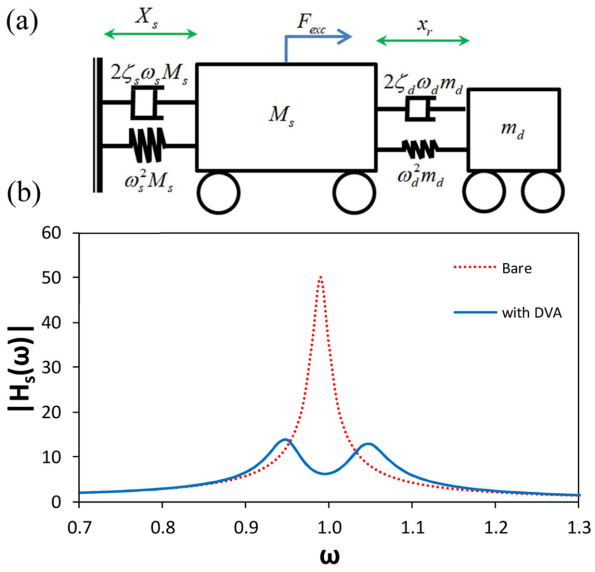


Figure 1. (a) structure-DVA system (b) mechanical admittance function.

2.2. Effective Damping

The performance of a DVA is characterized by the amount of "added effective damping", ζ_{add} that it appears to add to the structure. The "total effective damping", $\zeta_{eff} = \zeta_s + \zeta_{add}$ is the sum of the inherent structural damping and added effective damping. The total effective damping, ζ_{eff} relates the response variance of the structure with ($\sigma^2_{s-damped}$) and without (σ_s^2) the damping system (McNamara 1977):

$$\zeta_{eff} = \zeta_s \frac{\sigma_s^2}{\sigma^2_{s-damped}} \tag{5}$$

If the parameters of the system are known, the total effective damping may be computed as (McNamara 1977):

$$\zeta_{eff} = \frac{\omega_s}{2} \left(\frac{A_1(A_2A_3 - A_1A_4) - A_0A_3^2}{\frac{B_0^2}{A_0}(A_2A_3 - A_1A_4) + A_3(B_1^2 - 2B_0B_2) + A_1B_2^2} \right) \tag{6}$$

It is critical to recognize that the total effective damping of the system is the level of damping that would result in the (single degree of freedom) structure attaining the same response as that of the structure equipped with the DVA. Thus, the damping of the structure is increased until the response variance of this system is equal to the response variance of the structure-DVA system. This equivalence is demonstrated in Figure 2, where the area under the two curves (squared) is equal. However, while the variances of the structure-DVA system and the bare structure with

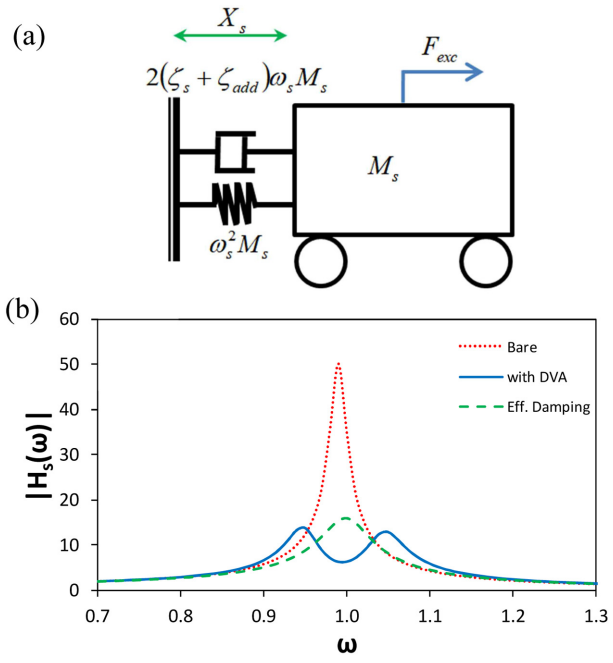


Figure 2. (a) Effective damping representation (b) mechanical admittance functions with effective damping.

higher effective damping are equal, the responses themselves (that is, the shape of the curves) are very different.

3. A False Start: Random Decrement and the DVA

Before describing methods that can be used to evaluate the DVA performance, it is first necessary to dispel a "false start" that the authors have seen employed by numerous researchers and practitioners. The random decrement technique (RDT) is a method that has been used for decades to estimate the inherent damping of uncoupled structures subjected to random excitation (Cole 1971). RDT generates an artificial free decay (or "RDT signature") of the structure by superimposing many segments of the measured structural response together after a specified trigger condition has been met. Although each individual response segment includes the effects of the random loading, when many segments are superimposed, the resulting structural response "averages out" the effects of the uncorrelated random loading, leaving only the free decay from the specified trigger level. Under certain conditions, the RDT signature can be generated using the autocorrelation of the measured response (Asmussen et al. 1998). The logarithmic decrement technique is then applied to estimate the structural damping by enveloping the RDT signature according to:

$$A(t) = A' \exp(-\zeta_s \omega_s t) \tag{7}$$

where A' is the specified trigger level or starting amplitude.

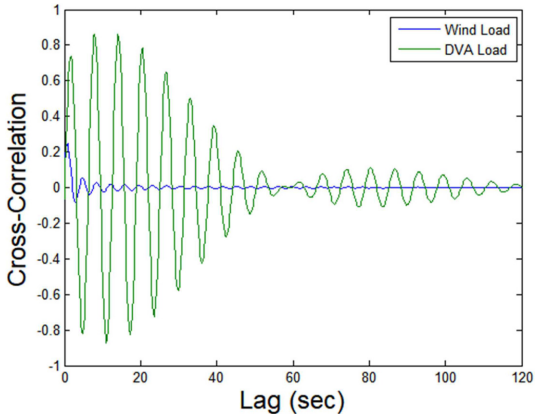


Figure 3. Cross-correlation of the wind and DVA loads with the structural acceleration.

RDT is developed from the equation of motion of a single degree of freedom system. It requires that all forces acting on the structure are uncorrelated with the structural response, which is achieved if the forces can be approximated as white noise excitation (Vandiver et al. 1982). However, for a DVA to operate properly, the forces that the DVA applies to the structure must be strongly correlated (although phase-lagged) with the structure's response. This strong cross-correlation is demonstrated in Figure 3 for the example system considered previously. Simulations are conducted in which the linear structure-TMD system is subjected to 44 hours of band-limited (0.3-1.7 rad/s) white noise excitation. The cross-correlation of the applied wind loading with the structural response is small; however, the cross-correlation between the DVA loading and the structural response is high. Therefore, the assumption that the loading will "average out" when many segments are superimposed is severely violated.

As an example, RDT is applied to a structure-DVA system to demonstrate its inadequacy. For the system considered previously, Eq (6). produces $\zeta_{eff} = 3.28\%$. RDT is applied to the simulations to develop the RDT signature shown in Figure 4(a). As is typical of structures equipped with DVAs, "beating" is observed in the free decay, since energy is transferred back-and-forth between the structure and DVA. The "correct" envelope, corresponding to $\zeta_{eff} = 3.28\%$, is plotted on the same axes. In this case, this envelope nearly bounds the "beats" in the RDT signature. While this may appear to support the use of RDT for DVA applications, this occurrence is not guaranteed or even typical. If the DVA damping was only $\zeta_d = 3.5\%$ (instead of the better-performing level of 5.0%), Eq. (6) reveals ζ_{eff} decreases slightly to $\zeta_{eff} = 3.15\%$. Figure 4(b) shows the RDT signature generated plotted alongside the corrected envelope ($\zeta_{eff} = 3.15\%$). The envelope does not bound the "beats", but rather passes through the RDT signature indicating that RDT cannot reliably predict or detect ζ_{eff} for structure-DVA systems.

Frequency-domain methods (such as the half-power

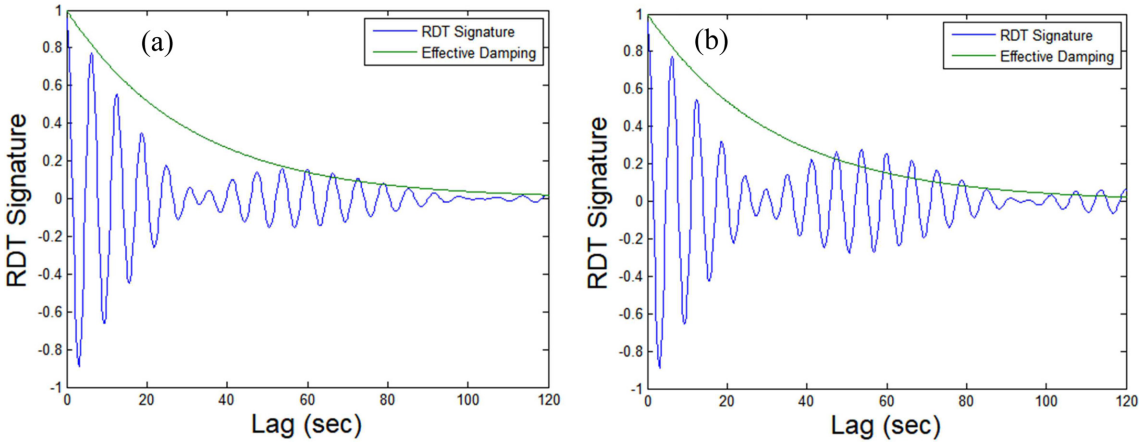


Figure 4. Example of erroneous application of RDT alongside known free decay envelopes (a) $\zeta_d = 5.0\%$, (b) $\zeta_d = 3.5\%$.

bandwidth method (Chopra 2007)) employed for estimating the damping of bare structures are also unsuitable for structure-DVA systems. Like RDT, these methods are derived from the equation of motion of an uncoupled single-degree-of-freedom system, and do not properly accommodate the dynamic interaction between the structure and DVA. The issues that plague RDT in the time domain also plague the half-power bandwidth method in the frequency domain.

4. Direct Measurement

Damping is proportional to the rate of energy removal from a dynamic system relative to the total energy in that system. To properly quantify the total energy in a structure-DVA system, the responses of both the structure and DVA must be measured synchronously. In this section, practical methods to quantify the added effective damping and the inherent structural damping of a structure-DVA system are described.

4.1. Added Effective Damping Estimation

When a structure is subjected to random white noise excitation, the added effective damping can be calculated as (Love and Tait 2017):

$$\zeta_{add} = \frac{\omega_s \mu E[\ddot{X}_s \dot{x}_r]}{2E[\ddot{X}_S^2]} \tag{8}$$

where $E[\ddot{X}_s \dot{x}_r]$ is the covariance between the measured structural acceleration and measured DVA velocity, and $E[\ddot{X}_S^2]$ is the variance of the measured structural acceleration. Since the kinetic energy of an oscillator is proportional to its mass, it is necessary that the DVA mass ratio, μ is known beforehand, and the structure's natural frequency, ω_s must also be known or determined from the structural monitoring. This method has been validated using numerical simulations, and scale-model testing (Love and Tait 2017), and has been applied to structural monitoring data obtained from several full-scale structure-TMD and structure-TSD systems (Love et al. 2018, 2020; Love and Haskett 2019).

Equation (8) provides a practical means to directly quantify the added effective damping, ζ_{add} provided by a DVA. To determine the total effective damping, ζ_{eff} the inherent structural damping, ζ_s must be determined as described in the next section.

4.2. Inherent Structural Damping Estimation

The simplest means to obtain ζ_s is to measure the ambient response of the structure prior to the DVA commissioning (that is, before the TMD mass is released or liquid is added to the TSD tank). The bare structure response enables ζ_s to be estimated using RDT. Unfortunately, damping estimation techniques often require modest to large wind events to ensure the structural response overcomes the measurement "noise", and captures any amplitude dependency of ζ_s (Tamura 2012). If a modest wind event is not captured by structural monitoring between the time of substantial construction completion and DVA commissioning, then it will be challenging to reliably estimate ζ_s using traditional methods.

A modified version of RDT has been proposed for structure-DVA systems subjected to random wind loading (Love and Haskett 2019). Like the traditional RDT, the method generates RDT signatures or "free decays" of the structure and DVA using cross-correlation functions. The RDT signature of the structure from a specified level is composed of a "free decay" component as the structural response decays from the specified trigger level, and a "forced" component arising from the loads imparted from the DVA on the structure. Mathematically, this can be represented as (Love and Haskett 2019),

$$D_X(t) = \exp(-\zeta_s \omega_s t)\cos(\omega_s t) + m_d \int_0^t h(t-\tau) D_{\dot{x}}(\tau) d\tau \tag{9}$$

$$h(t) = \frac{1}{\omega_s M_s} \exp(-\zeta_s \omega_s t)\sin(\omega_s t) \tag{10}$$

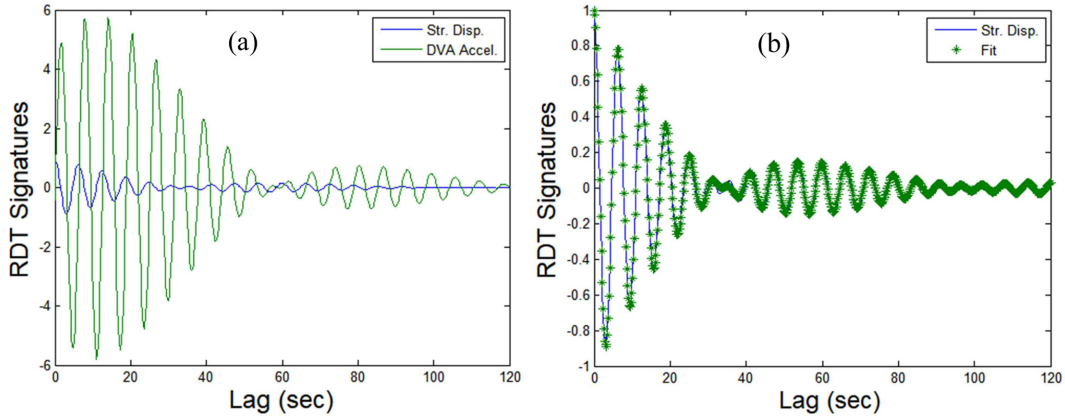


Figure 5. (a) RDT signature of structural displacement and DVA acceleration, (b) fit of structural displacement RDT signature $\zeta_s = 1.0\%$, $\omega_s = 1.0$ rad/s.

$$D_X(\tau) = -\frac{R_{\ddot{X}X}(\tau)}{E[\ddot{X}_s^2]} \tag{11}$$

$$D_{\ddot{x}}(\tau) = \frac{R_{\ddot{X}\ddot{x}}(\tau)}{E[\ddot{X}_s^2]} \tag{12}$$

where $D_X(t)$ is the RDT signature of the structural displacement, $D_{\ddot{x}}(\tau)$ is the RDT signature of the DVA acceleration, $h(t)$ is the impulse response function of the bare structure, $R_{\ddot{X}X}(\tau)$ is the cross-correlation function of structural acceleration with the structural displacement, and $R_{\ddot{X}\ddot{x}}(\tau)$ is the cross-correlation function of structural acceleration with the DVA acceleration. With the RDT signature for the DVA acceleration, $D_{\ddot{x}}(\tau)$ measured through DVA monitoring, Eq. (9) is used to determine the values of ω_s and ξ_s that produce the best fit to the RDT signature of structural displacement, $D_X(t)$. Figure 5(a) shows the RDT signatures for the simulated system considered in Section 2.1, and Figure 5(b) shows the fit to $D_X(t)$. The values $\omega_s = 1.0$ rad/s and $\zeta_s = 1.0\%$ provide an excellent fit to the data. The best fit values for ω_s and ζ_s can be determined using a numerical optimization routine, although an energy balance approach using linear regression has also been suggested (Love and Haskett 2019).

If synchronous measurements of the structural and DVA responses can be obtained, and μ is known, ζ_s and ζ_{add} can be directly predicted using the methodologies presented herein. These methods make no assumptions regarding the DVA tuning ratio, damping ratio, or even the form (e.g. linear, quadratic, etc.) of the DVA damping. They instead rely only upon the inertial force that the DVA applies to the structure.

5. System Identification

If measurements of the structural and DVA responses are not synchronous, it is not possible to quantify the flow of energy within the structure-DVA system. Instead, the performance of a DVA may be ascertained using system identification, whereby structural monitoring is used to estimate the dynamic properties of the structure and DVA. With the dynamic properties estimated, the theoretical performance of the DVA is calculated using Eq. (6). While several sophisticated algorithms have been proposed to identify the properties of structure-TMD systems (Hazra et al. 2010; Roffel and Narasimhan 2016), herein robust, practical methods are employed. Two forms of system identification are described: those relying upon ambient excitation, and those relying upon intentional excitation.

5.1. Ambient Monitoring

The dynamic properties of the structure and DVA may be estimated using monitoring data collected during ambient wind excitation. Since the excitation is random and unknown, the dynamic properties are estimated by comparing the measured response spectra of the structure and DVA to the predicted spectra. The dynamic properties of the system are adjusted until the predicted spectra match the measured spectra.

Since TSDs are frictionless systems, the liquid will respond under all amplitudes of structural excitation. However, TSD damping is nonlinear and will be very low on calm days. This low damping can be advantageous for system identification, since it results in clear peaks in the measured spectra, which can be readily aligned with peaks in the predicted spectra to confirm natural frequencies of the structure and TSD tank(s).

Figure 6 shows a stacked three-tank TSD system installed in a 70-storey building in Chicago, USA. The measured structural frequency is 1.07 rad/s. The key dimensions (length × width × water depth) of the three tanks are 12.0 m × 16.0 m × 1.45 m (bottom tank), 12.0 m ×16.0 m, × 1.83 m (middle tank), and 12.0 m × 9.45 m × 2.43 m (top tank). Each tank is tuned to a slightly different

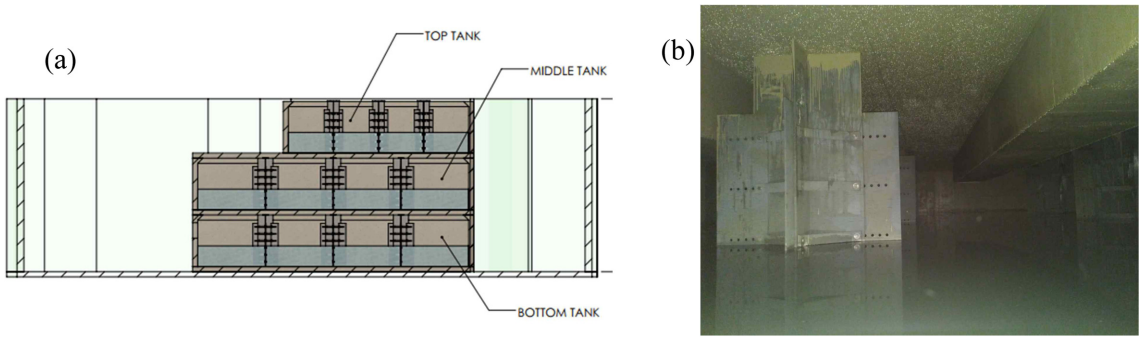


Figure 6. (a) Elevation view of stacked TSD tanks, and (b) photo of a tank interior.

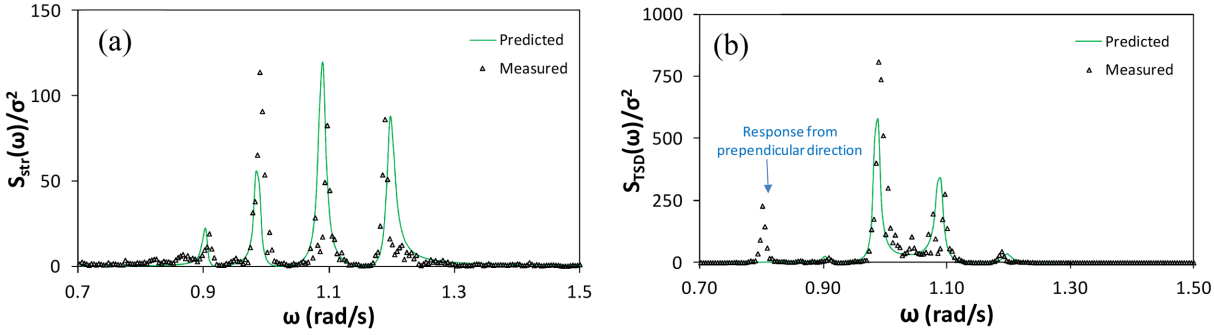


Figure 7. Measured and predicted spectral responses (a) structural acceleration, (b) middle tank wave height.

frequency, which maximizes the TSD performance and robustness (Love and Tait 2015). The TSD mass ratio of all tanks combined is 3.8%. TSD damping is provided by nine paddles (1.0-1.5m wide each) arranged in a 3 × 3 grid in each tank.

Figure 7 shows the spectra (normalized by variance) of the measured and predicted structural acceleration and TSD wave height of the middle tank. Since this system consists of three tanks with dissimilar natural sloshing frequencies, the structural acceleration spectrum contains four clear peaks close to 0.9, 1.0, 1.1, and 1.2 rad/s. Peaks at these frequencies are also visible in the wave height spectrum, although the peaks at 0.9 and 1.2 Hz are far less pronounced. A peak is also visible in the wave height spectra near 0.8 rad/s, however, this peak is due to sloshing in the perpendicular direction and is therefore neglected. The peaks predicted by the structure-TSD model (Love and Tait 2015) are in agreement with those measured, indicating the dynamic properties are accurately identified.

These measurements were conducted during very light winds, where the measured peak acceleration was ~0.4 milli-g, and the standard deviation was ~0.1 milli-g. For this level of excitation, the model predicts a total effective damping of ~2.1%, and the inherent structural damping ratio is estimated to be 1.0%. As the excitation amplitude increases, the TSD damping ratio will increase closer to its optimal value, and the total effective damping will approach 6.0%, as indicated in Figure 8.

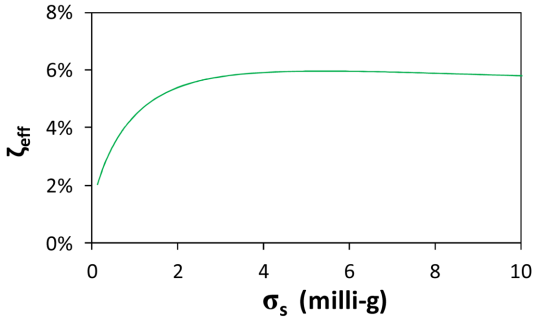


Figure 8. Predicted total effective damping at various initial (uncontrolled) RMS structural acceleration amplitudes.

System identification and verification through ambient monitoring works well for TSDs, which are frictionless and will respond at all levels of structural excitation. Conversely, TMDs are mechanical systems that will possess some friction that must be overcome before it responds relative to the structure. Good design practice will result in friction forces that are generally overcome at structural acceleration levels of ~0.5 milli-g, which is well below the threshold of human perception. However, during short-term ambient vibration monitoring, it is unlikely that structural accelerations will attain 0.5 milli-g, since it is unlikely that moderate winds will occur during the monitoring period. Moreover, even if moderate wind events occur and accelerations of a few milli-g are attained, the effects of friction can confound monitoring data

interpretation, since the TMD will be "free" and "locked" during different parts of the wind event when the building response is large or small. For this reason, pull tests are typically required to verify the TMD performance, as described in the next section.

5.2. Pull Tests

Pull tests are generally required to identify or confirm the dynamic properties of structure-TMD systems for performance verification. For these tests, the TMD mass is manually displaced using a temporary system that offers high mechanical advantage, and then is suddenly released. After release, the TMD will oscillate, transmitting energy to the structure, which will in turn begin to oscillate. Energy will continue to be transmitted back-and-forth between the structure and TMD as the system response decays. The accelerations of the structure and TMD are measured, and the measured response is compared to the predicted response to confirm system parameters.

Figure 9 shows an example of an opposed-pendulum style of TMD (Morava et al. 2012) which is installed in a 71-storey tower in New York City, USA. The measured tower natural frequency is $\omega_s = 0.702$ rad/s, and the TMD mass ratio is 4.4%. A suite of pull tests was conducted at several initial TMD displacement amplitudes. For each pull test, numerical optimization is used to determine the structure and TMD properties that provide a best fit to the measured structure and TMD responses. Figure 10 (a) and (b) show the measured and best fit responses for tests that were conducted with the TMD's viscous damping

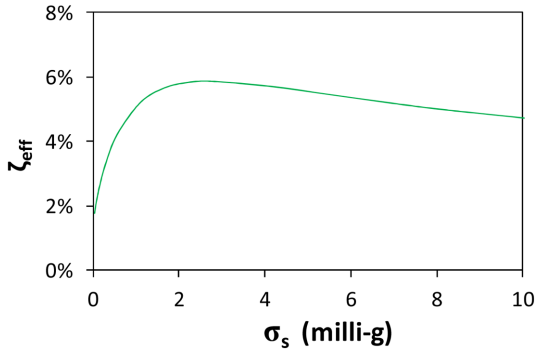


Figure 11. Predicted total effective damping at various initial (uncontrolled) RMS structural acceleration amplitudes.

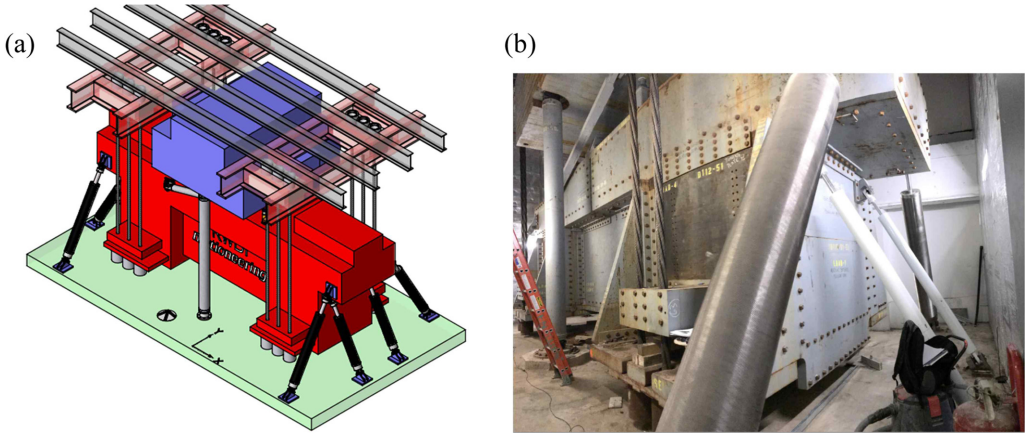


Figure 9. TMD (a) schematic, and (b) photo.

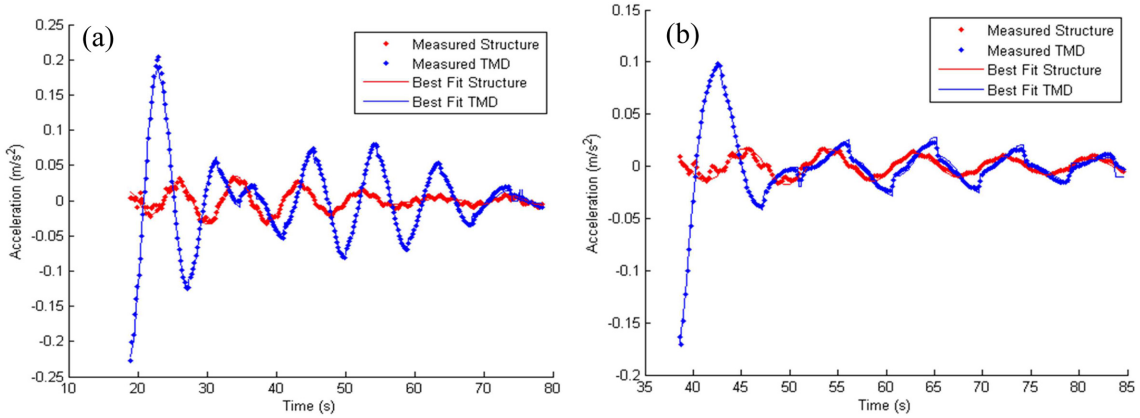


Figure 10. Pull test at 0.4m initial TMD amplitude (a) VDDs disconnected, (b) VDDs connected.

devices (VDDs) disconnected, and connected, respectively. As expected, the TMD and structural response amplitudes are larger and last longer when the VDDs are disconnected. With the system properties identified, ζ_{eff} can be predicted for various levels of uncontrolled structural acceleration amplitudes as shown in Figure 11.

6. Conclusions

Full-scale performance verification of dynamic vibration absorbers has received little attention in the literature. In this paper, the errors associated with using techniques intended for uncoupled systems, such as RDT, are described and illustrated with examples. To properly assess DVA performance, structural monitoring should measure the response of the structure and DVA so that the transmission and dissipation of energy within the structure-DVA system can be quantified. Ideally, synchronous measurements of the structure and DVA responses are conducted to allow the added effective damping provided by the DVA and inherent structural damping to be directly measured. If the measured responses are not synchronous, structural monitoring is conducted to identify or verify the structure and DVA properties and then the total effective damping is estimated using theory. Ambient monitoring is typically used for TSD systems, whereas pull tests are often employed for TMD systems. Through the proper use of structural monitoring data, the performance of DVAs installed to improve the motion comfort of tall buildings can be verified.

References

Asmussen, J. C., Ibrahim, S. R., and Brincker, R. (1998). "Random decrement: Identification of structures subjected to ambient excitation." *Proceedings of the International Modal Analysis Conference - IMAC*, 914-921.

Caughey, T. K. (1963). "Equivalent Linearization Techniques." *The Journal of the Acoustical Society of America*, 35(11), 1706-1711.

Chopra, A. K. (2007). *Dynamics of Structures: Theory and Applications to Earthquake Engineering*. Pearson Prentice Hall, Upper Saddle River, NJ.

Cole, H. A. (1971). *Failure detection of a space shuttle wing flutter model by random decrement*. *NASA-TM-X-62041*, Moffett Field, California.

Hazra, B., Roffel, A. J., Narasimhan, S., and Pandey, M. D. (2010). "Modified Cross-Correlation Method for the Blind Identification of Structures." *Journal of Engineering Mechanics*, 136(7), 889-897.

Love, J. S., and Haskett, T. C. (2019). "Measuring inherent structural damping of structure-TMD systems." *Engineering Structures*, Elsevier, 196(June), 109300.

Love, J. S., Haskett, T. C., and Morava, B. (2018). "Effectiveness of dynamic vibration absorbers implemented in tall buildings." *Engineering Structures*, Elsevier, 176 (September), 776-784.

Love, J. S., Morava, B., and Smith, A. W. (2020). "Monitoring of a Tall Building Equipped with an Efficient Multiple-Tuned Sloshing Damper System." *Practice Periodical on Structural Design and Construction*, 25(3), 05020003.

Love, J. S., and Tait, M. J. (2015). "Multiple Tuned Liquid Dampers for Efficient and Robust Structural Control." *Journal of Structural Engineering*, 141(12), 04015045.

Love, J. S., and Tait, M. J. (2017). "Estimating the added effective damping of SDOF systems incorporating multiple dynamic vibration absorbers with nonlinear damping." *Engineering Structures*, Elsevier Ltd, 130, 154–161.

McNamara, R. J. (1977). "Tuned Mass Dampers for Buildings." *Journal of the Structural Division*, 103(9), 1785-1798.

Morava, B., Haskett, T., and Smith, A. (2012). "Enhancing the serviceability performance of tall buildings using supplemental damping systems." *Ingegneria Sismica*, 29(1), 60-70.

Roffel, A. J., and Narasimhan, S. (2016). "Results from a Full-Scale Study on the Condition Assessment of Pendulum Tuned Mass Dampers." *Journal of Structural Engineering*, 142(1), 04015096.

Rüdinger, F. (2007). "Tuned mass damper with nonlinear viscous damping." *Journal of Sound and Vibration*, 300(3-5), 932-948.

Tait, M. J. (2008). "Modelling and preliminary design of a structure-TLD system." *Engineering Structures*, 30(10), 2644-2655.

Tamura, Y. (2012). "Amplitude Dependency of Damping in Buildings and Critical Tip Drift Ratio." *International Journal of High-Rise Buildings*, 1(1), 1-13.

Vandiver, J. K., Dunwoody, A. B., Campbell, R. B., and Cook, M. F. (1982). "A Mathematical Basis for the Random Decrement Vibration Signature Analysis Technique." *Journal of Mechanical Design*, 104(2), 307-313.

Warburton, G. B. (1982). "Optimum absorber parameters for various combinations of response and excitation parameters." *Earthquake Engineering & Structural Dynamics*, 10(3), 381-401.