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Sensitivity Analysis Related to Redundancy of Regular and Irregular Framed Structures after Member Disappearance

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Abstract

Recently, there have been some reported examples of structural collapse due to gravity, subsequent to damage from accident or an excitation that was not prepared for in the design process. A close view of new concepts, such as a redundancy and key elements, has been taken with the aim of ensuring the robustness of a structure, even in the event of an unexpected disturbance. The author previously proposed a sensitivity index of the vertical load carrying capacity to member disappearance for framed structures. The index is defined as the ratio of the load carrying capacity after a member or a set of an adjacent member disappears, to the original load carrying capacity. The member with the highest index may be regarded as a key element. The concept of bio-mimicry is being applied to various fields of engineering, and tree-shaped structures are sometimes used for the design of building structures. In this study a sensitivity analysis is applied to the irregular-framed structures such as tree-shaped structures.

Keywords: Vertical load carrying capacity, Sensitivity index, Key element, Framed structures, Tree-shaped structure

1. Introduction

Recently, there have been some reported examples of structural collapse of the building structures due to gravity, subsequent to damage from an accident or an excitation that was not prepared for in the design process. A close view of new concepts, such as a redundancy and key elements, has been taken with the aim of ensuring the robustness of a structure, even in the event of an unexpected disturbance.

Herein, after the member is disappeared, one of the worst scenario is the progressive collapse, and the most under researched areas in structural engineering. Furthermore, a lot of literature and the preparation for the design code against progressive collapse are released in these days (Marco and Uwe, 2009; Osama, 2006; AIJ, 2013).

These publications are categorized into three themes; progressive collapse potential, progressive collapse resistance, and the case study on actual structures (AIJ, 2013). For instance, Fragopol (1987) and Ito (2005), etc. present the index related to redundancy and key-element. Kim (2009) and Ito (2011, 2012), etc. studied the progressive collapse potential with catenary action during a large deformation occurrence. Furthermore, the analytical studies related to redundancy and progressive collapse on the actual building structure were conducted by Ohi (2007) and Kim(2012), etc.

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The past research of the index related to redundancy or key-elements are as follows; Fragopol et al. (1987), Wada et al. (1989), Feng et al. (1986), Schafler et al. (2005), and Paliou et al. (1990) all studied how much resistance a structure would retain after components of the structure had been destroyed by accidental action, and compared this to the resistance in the original state. Furthermore, we proposed the sensitivity analytical method related to structural redundancy (Ito et al., 2005). This method estimates a sensitivity index that represents the lack of resilience of the vertical load carrying capacity to member disappearance, based on plastic analysis. The index is defined as the ratio of the load carrying capacity after a member disappears to the original load carrying capacity. Fig. 1 shows a conceptual diagram of the estimation method.

Fig. 2 shows a design flow chart for structures with high redundancy that has been proposed (JSSC, 2005). In this design procedure, a key element is chosen by reference to the sensitivity index (Ito et al., 2005). Additionally, a design code or guideline for structures with high redundancy is currently being prepared (Marco and Uwe, 2009; Osama, 2006).

The concept and the characteristics of bio-mimicry are becoming increasingly focused on in various fields of engineering technology. A tree-shaped structure is one such form of building that uses bio-mimicry, and has already seen practical application, for example in Gaudi's Sagrada Família, the Stuttgart Airport terminal building, etc. Saito et al. (2006) studied the structural characteristics of tree-shaped structures. In the present work, the redundancy of irregular-framed structures, such as tree-shaped struc-

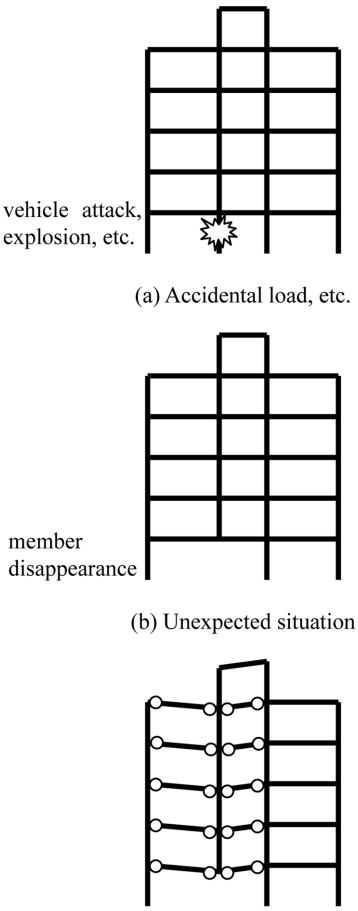


Figure 1. Conceptual diagram of estimation method related to redundancy.

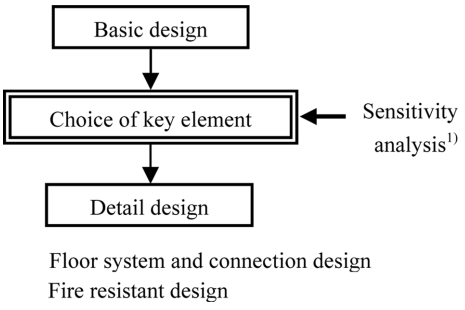


Figure 2. Design flow to building structure with high redundancy (JSSC, 2005).

tures, is studied by means of a sensitivity analysis.

2. Sensitivity Analysis Method

2.1. Sensitivity index

The author proposed a sensitivity index for framed structures, representing the lack of resilience of the vertical load carrying capacity to member disappearance (Ito et al., 2005). The index is defined as the ratio of the load carrying capacity after a member disappears to the original load carrying capacity. The sensitivity index is denoted by *S.I.*:

$$S.I. = \frac{\lambda_0 - \lambda_d}{\lambda_0} \tag{1}$$

Where, λ_0 is the collapse load factor in the original state, and λ_d is the collapse load factor of a frame after the disappearance of a member.

The above formula is equivalent to the reciprocal of the redundancy index given by Fragopol et al. (1987). When the vertical load carrying capacity is unaffected by the disappearance of a certain member, then the corresponding sensitivity index is very small ($S.I. \approx 0$). Such a member does not control the load carrying capacity of the whole structural system, and would be regarded a less important from the standpoint of reserving the load carrying capacity. Conversely, when a member with large sensitivity, ($S.I. \approx 1$) disappears, a part of the frame or the whole frame would collapse immediately. Thus, such a member with a high sensitivity index may be regarded as a key element of the structural system (Ito et al., 2005).

2.2. Matrix method of limit analysis

This paper makes use of the matrix method proposed by Livesley (1976) termed "Compact procedure", which is based on the lower-bound theorem for the limit analysis and linear programming for the optimization problem. The compact procedure solves the following problem:

$$\text{Maximize} \quad \lambda \tag{2}$$

$$\text{Subject to} \quad \lambda\{P_0\} = [\ Con.\ ] \cdot \{M\} \tag{3}$$

$$|M_j| \le M_{Pj} \tag{4}$$

where $\{P_0\}$ is the nodal load vector, [*Con.*] is the connectivity matrix in equilibrium equation, $\{M\}$ is the member force vector, and M_p is the plastic resistant moment of a member.

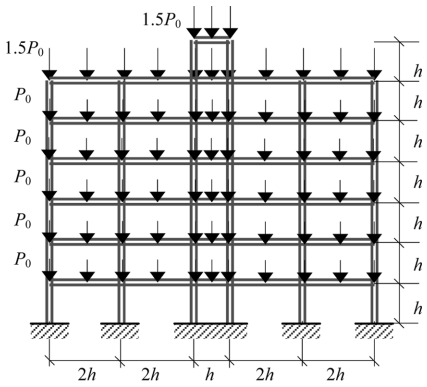
2.3. Modification of equilibrium equation

In the case that a certain member in the frame disappears, the constraints represented by Eqs. (3) and (4) are modified. The force of the member that disappears is simply removed from the member force vector, and the corresponding column of the connectivity matrix is also removed. A computer program can perform this automatically.

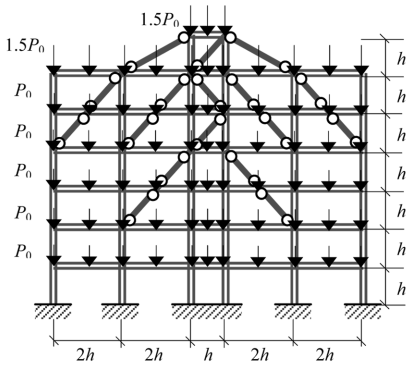
$$\lambda_d \begin{Bmatrix} P_0 \end{Bmatrix} = \begin{bmatrix} C_{11} & \cdots & C_{1j} & \cdots & C_{n1} \\ \vdots & & \vdots & & \vdots \\ C_{m1} & \cdots & C_{mj} & \cdots & C_{nm} \end{bmatrix} \begin{bmatrix} M_1 \\ \vdots \\ M_j \\ \vdots \\ M_m \end{bmatrix}$$

remove (column $C_{1j} \cdots C_{mj}$)

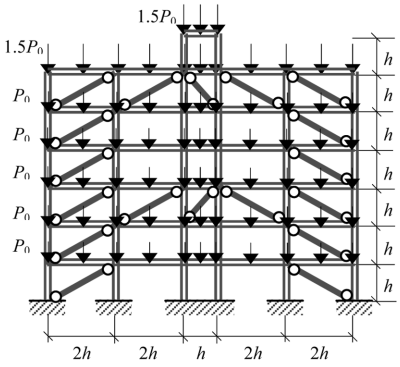
remove (M_j)



(a) Moment resisting frame



(b) Frame with outrigger truss



(c) Megastructure

Figure 3. Analytical model of multi-story planar frame.

3. Example of Sensitivity Analysis for Multi-story Framed Structures

3.1. Summary of analytical study

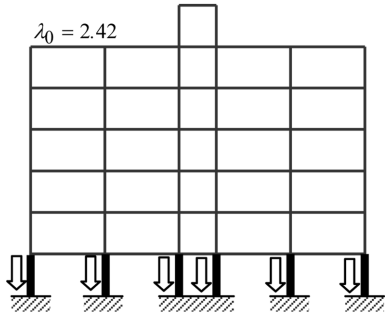
In this section, the sensitivity analysis is explained and an example 7-story 5-bay planner frame (shown in Fig. 3) is studied. Furthermore, the structural forms of the framework are considered as analytical parameters, such as an outrigger truss and a mega-structure. These analytical models are applied to the sensitivity analysis.

3.2. Analytical model and parameters

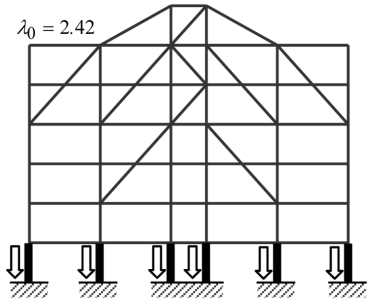
The frame has a story height h and span length $2h$ (h for the central short span length). The column over-design factor (or the beam-to-column moment capacity ratio) of each story is 1.2, representing a weak-beam structure. The plastic moment capacity of the beams of floors two through four is $1.2M_0$, or 1.2 times the fully plastic mo-

Table 1. Plastic resistance of members

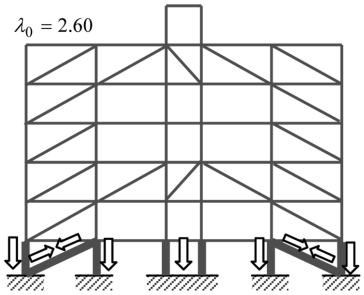
Inter-story	Floor	Member	Flexural Resistance	Axial Resistance
-	2-4	Girder	$1.2M_0$	25.6 M_0/h
1-3	-	Column	$1.44M_0$	30.7 M_0/h
3	-	Truss	-	12.2 M_0/h
1-3	-	Truss	-	12.2 M_0/h
-	5-PHR	Girder	M_0	21.3 M_0/h
4-7	-	Column	$1.2M_0$	25.6 M_0/h
4-7	-	Truss	-	12.2 M_0/h



(a) Moment resisting frame

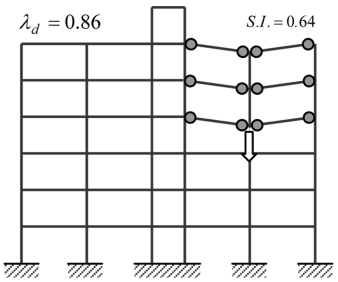


(b) Frame with outrigger truss

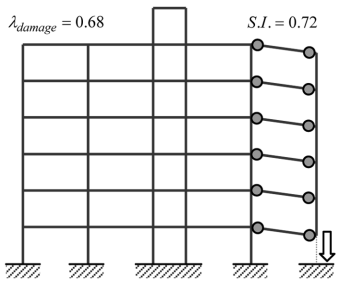


(c) Megastructure

Figure 4. Limit state analysis on original state.

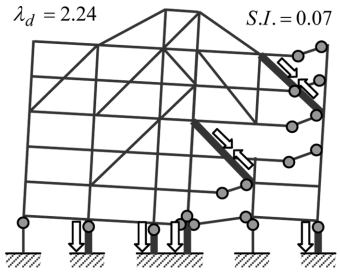


(a) Fourth story mid column disappearance

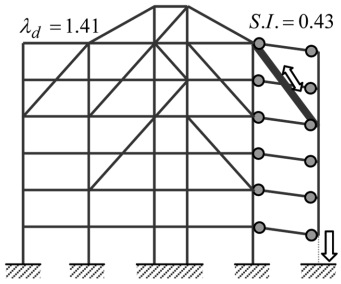


(b) First story side column disappearance

Figure 5. Sensitivity analysis on frame.



(a) Fourth story mid column disappearance



(b) First story side column disappearance

Figure 6. Sensitivity analysis on frame with truss.

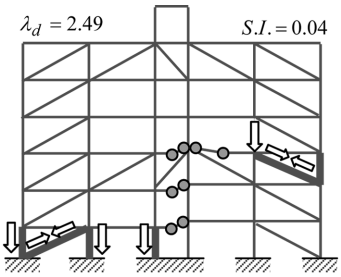
ment M_0 of the beams of floors five through to the penthouse roof. The relationship $N_y = 64M_{p0}/3h$ is assumed for the plastic resistance, but the interaction between the axial and flexural plastic resistances is ignored.

The buckling of members is not considered. For the plastic capacity, each truss member is assumed to have pinned joints at both ends, and to be strong enough that the lateral component of resistance exceeds (by a factor of three) the story-shear resistance expected in the surrounding moment resisting frame. Table 1 summarizes the plastic resistances of the members. Vertical loads are applied to each column top and each beam center as concentrated loads. The magnitude of the vertical loads at the roof floor and the penthouse roof floor are $1.5P_0$, i.e., 1.5 times the standard magnitude P_0 of the load at the other floors. Fig. 4 shows the vertical load carrying capacity in the original state for three types of frame. It can be seen that the load carrying capacity in the original state is almost identical for each type of frame.

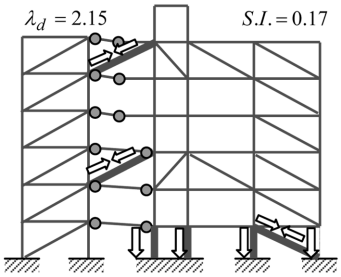
3.3. Results of sensitivity analysis and observations

The sensitivity indices *S.I.* against the disappearance of the first side column and fourth mid-column are calculated, and the results are shown in Fig. 5. In addition, the collapse load factors and the corresponding collapse mechanism are united and shown in this figure. The thick line indicates axial yielding, and the circle indicates a plastic hinge.

From a comparison of Figs. 5 and 6, the sensitivity to column disappearance is smaller in the case of a frame with a truss installed in the upper levels than it is in a frame consisting only of flexural members, beams and columns. Therefore, a truss of this kind raises the redundancy of a moment-resisting frame against accidental action. Furthermore, from a comparison of Figs. 5 and 7, the sensitivity index of a megastructure is smaller than the sensitivity.

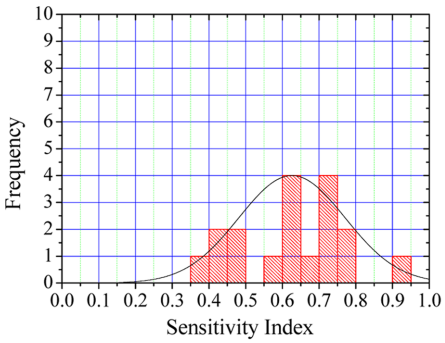


(a) Fourth story mid column disappearance

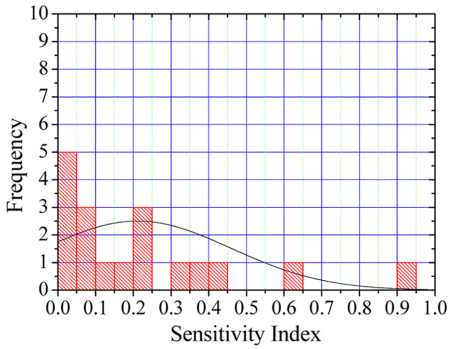


(b) First story side column disappearance

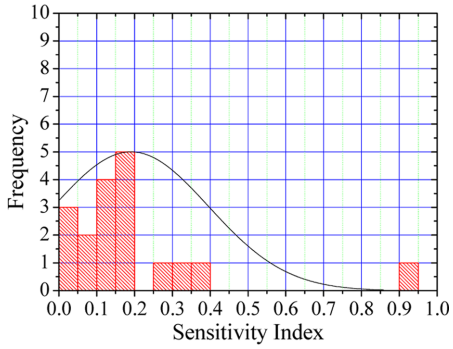
Figure 7. Sensitivity analysis on mega frame.



(a) Moment resisting frame



(b) Frame with outrigger truss



(c) Megastructure

Figure 8. Frequency distribution of sensitivity index.

When a column disappears in a moment resisting frame, a local collapse mechanism forms in the upper stories. Conversely, in a frame with a truss or a megacolumn installed, these members resist the descent of the local collapse mechanism, and prevent an abrupt drop in the load carrying capacity.

In the case that the column located in the middle of the frame disappears, for a frame consisting of only flexural members, the upper part of the frame descends considerably at the position of the column that disappeared, and a collapse mechanism is formed. However, for a frame with a truss in the upper levels, the presence of the truss prevents an abrupt drop in the load carrying capacity of the frame by means of the stress re-distribution effect.

Fig. 8 shows the frequency distributions of the sensitivity index after disappearance of one member. From Fig. 8, it can be seen that the moment-resisting frame has many members with high sensitivity.

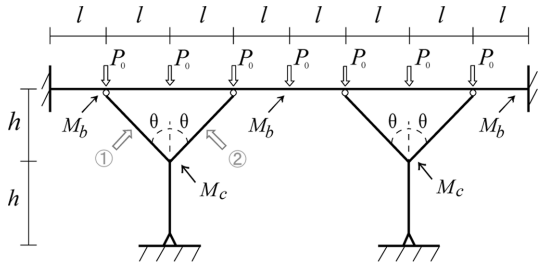


Figure 9. Single layer model.

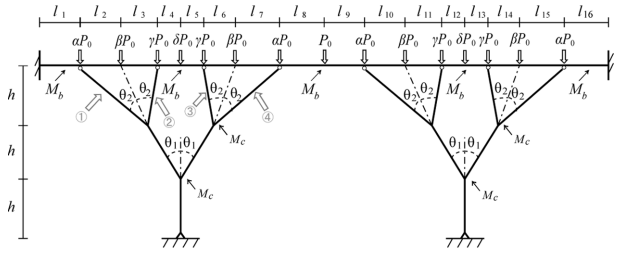


Figure 10. Double layer model.

4. Sensitivity Analysis on Irregular Framed Structures: Tree-shaped Structure

4.1. Summary of analytical study

Recently, attention has been given to the concept of bio-mimicry, and attempts have been made to introduce it into various fields of engineering technology, such as medical and electrical appliances. Within structural engineering, various types of innovative structural systems inspired by bio-mimicry are applied to building structures, such as honeycomb, cobweb, and tree-shaped structures (Frei Otto, 1986).

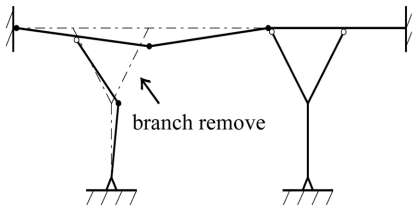
There have been few reports regarding elementally analytical study, the effect of geometrical configuration, and resistance mechanisms of tree-shaped structures (Saito et al., 2006).

According to these reports, a large branching angle could serve to mitigate the stress on the members and the displacement.

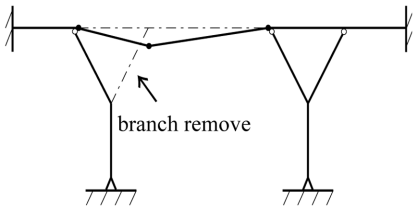
A sensitivity analysis for the tree-shaped structure shown in Figs. 9 and 10 is conducted. The parameters are the number of layers, the branching angle, and the strength of the members.

4.2. Sensitivity analysis of single-layer model

A preliminary study of sensitivity analysis on tree-shaped structures is conducted, in order to discuss the resistance mechanism and structural potential perform-

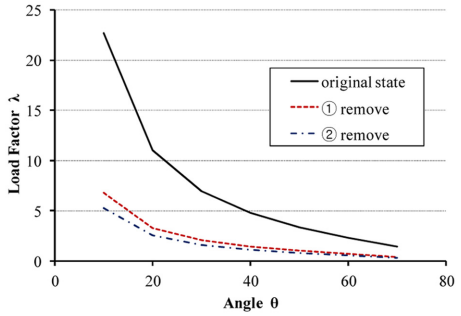


(a) Girder and column collapse mode

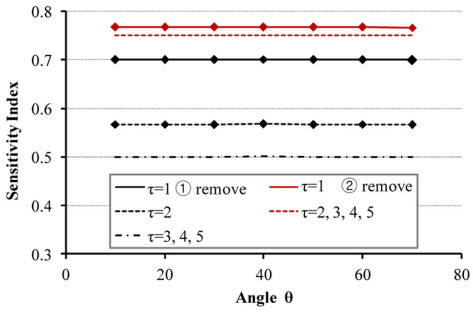


(b) Girder collapse mode

Figure 11. Collapse mechanism after a branch disappearance.



(a) Relation of load factor-the branching angle



(b) Relation of S.I.-the branching angle

Figure 12. Results of sensitivity analysis.

ance related to redundancy. Therefore, it is assumed that the sensitivity index to the slanting branch indicated in Fig. 9 is discussed. The disappearance members are also illustrated in Fig. 9.

The vertical loading condition is a uniformly distributed load, with intensity denoted by *P*. This is applied on the girder as shown in Fig. 9, and is replaced for modeling by a set of concentrated loads on the center of each girder and the branch member top. Both ends of girders are fixed as rigid.

The branching angle and the strength of members are considered as parameters in the study. The branching angle is varied from 10° to 70°, and the strength ratios of the column and branch to that of the girder are varied from 1 to 5.

Fig. 11 shows an example of the collapse load factor and the sensitivity index, after the disappearance of a single branch member from the original frame, divided by the combination of analytical parameters. Furthermore, Fig. 12 shows the results of the sensitivity analysis that relates the load factor and sensitivity index.

From the results of Fig. 12, it is confirmed that the load factor decreases as the branching angle increases, however, the sensitivity index indicates the same value, regardless of the branching angle. The reason for this is that the sensitivity index expresses the relative ratio of ultimate load carrying capacity before and after a member disappearance, so the sensitivity index will be unchanged if a failure mode such as girder failure occurs even after member disappearance.

4.3. Sensitivity analysis of double layer model

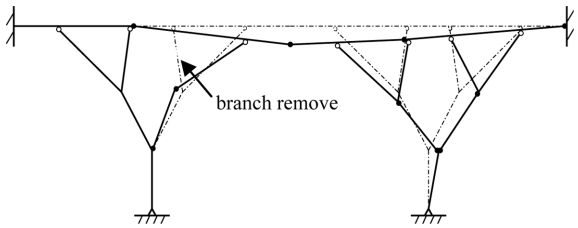
A sensitivity analysis is performed on a 2-layer tree-shaped model, in which two of the preliminary models described in the previous section are stacked, as shown in Fig. 10. The branching angles of each layer are considered as parameters; the angle of the first layer is constant at 30°, and the angle of the second layer is varied from 20° to 40°. Additionally, the strength ratios of the column and branch members to the girder are varied from 1 to 5. Both ends of the girders are fixed as rigid. The disappearance of members is illustrated in Fig. 10.

Fig. 13 shows an example collapse mechanism after a single branch member is removed from the original frame. Fig.14 shows the results of the sensitivity analysis as represented by the sensitivity index.

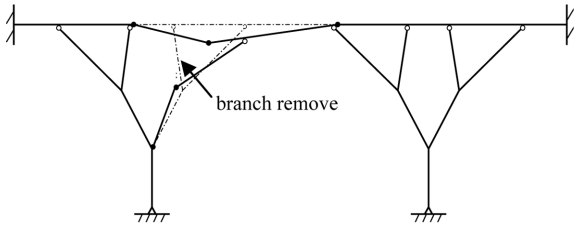
From the results of the sensitivity analysis, the failure modes after member disappearance are categorized into three modes as shown in Fig. 13 indicating the influence of the entire frame. Furthermore, Fig. 13(b) and 13(c) indicate the local collapse modes.

From the results of Fig. 14, it is confirmed that the sensitivity index decreases as the branching angle of second layer increases. This is because in case of a small branching angle, the vertical force at the node that is produced from the strength of the branch member becomes small. Consequently, the failure mode as shown in Fig. 13(a) occurs, and the load factor following member disappearance with a large second layer branching angle becomes large. That is, the decreasing ratio of load factor remains small.

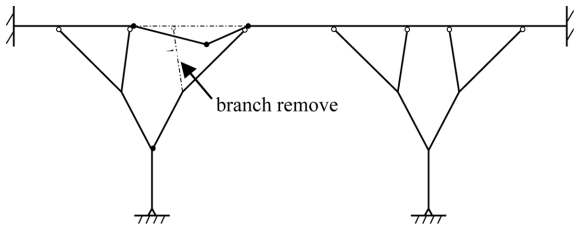
Furthermore, it is observed that the sensitivity index becomes zero after the strength ratio exceeds a certain value. This is because the same failure mode occurs, such as a beam collapse mode, and the ultimate strength after



(a) Girder branch failure (collaboration effect)



(b) Girder and branch failure (local collapse)



(c) Girder failure (local collapse)

Figure 13. Collapse mechanism after a branch disappearance.

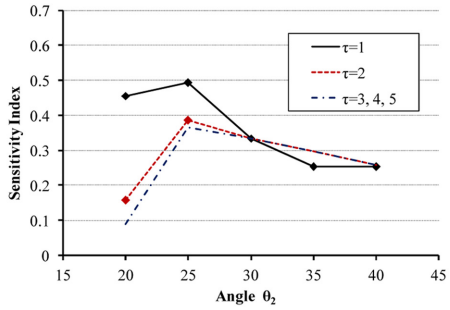
a member disappearance is the same as in the original state.

5. Concluding Remarks

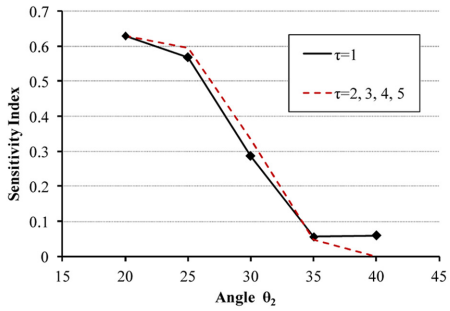
The author has proposed the sensitivity index related to redundancy (Ito et al., 2005). The method estimates the sensitivity of the vertical load carrying capacity to member disappearance, based on plastic analysis. The index is defined as the ratio of the load carrying capacity after a member disappears to the original load carrying capacity. In this study, a sensitivity analysis of two types of framed structure was conducted, a regular multi-story frame with and without a truss system or megastructural system, and a tree-shaped structure. From these case studies, the following conclusions can be drawn regarding the redundancy of framed and tree-shaped structures:

Multi-story regular frames were examined, with and without a truss system in the upper levels or a megastructure. A truss system or megastructural system effectively prevent the descent of upper portions, and lower the sensitivity of the frame to column disappearance.

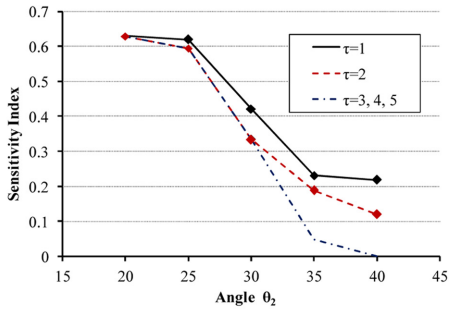
Tree-shaped structures were examined, with the branching angle and strength ratio of members. For multi-layer frames with a certain branching angle, the collaboration effects of the entire frames can prevent a large sensitivity index. This implies that tree-shaped structures have a potential to realize a structural system with high redundancy.



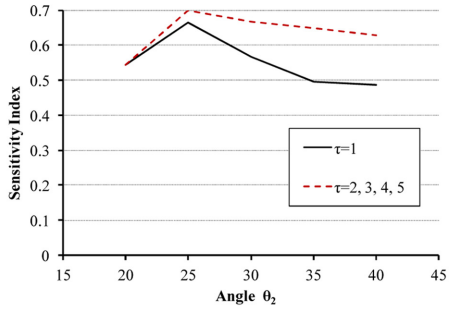
(a) Branch ① disappearance



(b) Branch ② disappearance



(c) Branch ③ disappearance



(d) Branch ④ disappearance

Figure 14. Results of sensitivity analysis (represented by *S.I.*- angle of branch).

References

Architectural Institute of Japan. (2013). Redundancy and Robustness in Building Structural Design.

Feng, Y. S. and Moses, F. (1986). "Optimum design, redundancy and reliability of structural systems", *Computers and Structures*, 24, pp. 239~251.

Frei Otto. (1986). Natural Structure, Kajima Institute Publishing Co. Ltd.

Fragopol, D. M. and Curley J. P. (1987) "Effects of Damage and Redundancy on Structural Reliability", *Journal of Structural Engineering, ASCE*, 113(7), pp. 1533~1549.

Ito, T. and Fukuyama, T. (2012). "Ultimate Behavior and Potential Resistant Mechanism of Framed Structures after Member Disappearance Subjected to Vertical Load," *Transactions of Structural and Construction Engineering*, 678, pp. 361~368. (in Japanese)

Japan Society of Steel Construction, JSSC. (2005). Collapse Control Design Recommendations for High-rise Steel Building Structure That Excels in Redundancy.

Choi, J. H., Ito, M., and Ohi, K. (2007). Prevention of Building Structural Collapse caused by Accidental Events, *Proc. of 2nd International Symposium on Improvement of Structural Safety for Building Structures*, pp. 85~98.

Kwon, K. H., Park, S. R. M., and Kim, J. K. (2012). "Evaluation of Progressive Collapse Resisting Capacity of Tall Building", *International Journal of High-Rise Buildings*, 1(3), pp. 229~235.

Kim, J. and An, D. (2009). "Evaluation of Progressive Collapse Potential of Steel Moment Frames Considering Catenary Action", *The Structural Design of Tall and Special Buildings*, 18, pp. 455~465.

Livesley, R. K. (1976). Matrix Method of Structural Analysis (2nd ed.), Pergamon Press.

Marco, H. and Uwe, S. (2009). Progressive Collapse Nomenclature, Structures 2009, Don't Mess with Structural Engineers, ASCE, pp. 1886~1895.

Nobuyuki, N., Miyasato, N., Saito, M. et al. (2006) Basic Study on Structural Morphogenesis of Tree-shaped Structure, Architectural Institute of Japan Kanto Branch Research Report Collection 1, 76, pp. 201~204 (in Japanese)

Osama, A. M. (2006). Progressive Collapse of Structures, Annotated Bibliography and Comparison of Codes and Standards, *Journal of Performance of Constructed Facilities, ASCE*, pp. 418~425.

Paliou, C., Shinozuka, M., and Chen, Y. N. (1990). "Reliability and redundancy of offshore structures", *Journal of Engineering Mechanics, ASCE*, 116, pp. 359~378.

Schafer, B. W. and Bajpai, P. (2005). "Stability degradation and redundancy in damaged structures", *Engineering Structures*, 27, pp. 1642~1651.

Ito, T., Ohi, K., and Li, Z. (2005). "A Sensitivity Analysis Related to Redundancy on Framed Structures Subjected to Vertical Loads," *Transactions of Structural and Construction Engineering*, 593, pp. 145~151. (in Japanese)

Ito, T. and Fukuyama, T. (2011). "A Potential Strength and Ultimate Behavior of Framed Structures Considering Catenary Effects after Failure Mechanism Formation Subjected to Vertical Load", *Theoretical and Applied Mechanics Japan*, 59, pp. 29~38.

Wada, A. et al. (1989) "A Study on Strength Deterioration of Indeterminate Double-layer Space Truss due to Accidental Member Failure", *Transactions of Structural and Constructional Engineering (Transactions of AIJ)*, 402, pp. 89~99. (in Japanese)