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Revisiting the "T" in TMD

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Abstract

This paper explores the meaning and importance of tuning amongst other Tuned Mass Damper (TMD) parameters and describes processes to help ensure that an as-built TMD is properly tuned to the as-built high-rise building. A summary of key TMD components and TMD implementations will be presented as an introduction and review. Next, it will be shown that tuning is a means for optimizing TMD performance. A process using modal characterization tests during tower construction to estimate natural frequencies of the completed tower will be described. Finally, the use of a Frequency Response Function (FRF) as a means for verifying the frequency of a TMD will be proposed.

Keywords: Tuned mass damper, Modal characterization, Tall buildings, Occupant comfort, Damping, Frequency

1. Introduction

Occupant comfort is a key consideration in the design of tall slender buildings, with slenderness ratios above about 10 : 1, subject to wind excitation. A common solution to improving such occupant comfort issues is to add damping. There are numerous ways to add damping, including distributed damping, slosh tanks, and liquid-column dampers. A Tuned Mass Damper (TMD) represents the most efficient use of the generally limited floor area at the top of a building and provides higher levels of effective damping over a broad range of tower motion.

When excessive tower motion is predicted to cause occupant discomfort in a slender tower, one or both of the two "sway" modes is the likely contributor. Each mode of vibration can be simplified as a single-degree-of-freedom (SDOF) system. A SDOF is shown in Figure 1a, where the tower's effective modal mass, M_0 is connected by a spring, K_0 and damper, C_0 to a fixed point. The modal mass of mode k is simply the sum of mass, m times modal displacement, ϕ squared over all floors, i and all degrees of freedom, j:

$$m_{jk}^{*}=\sum_{i} m\phi_{ijk}^{2} \tag{1}$$

The effective modal mass is then calculated at some location, such as the location of a TMD, by dividing the modal mass, m^* by the modal displacement squared at that location.

$$M_0=m^{*}/\phi^{2} \tag{2}$$

Given modal mass, natural frequency and an estimate of inherent damping, the spring and damping rates of the primary structure can be calculated.

Adding a TMD to increase damping in a mode of vibration can be represented as the addition of one degree of freedom to the SDOF (Figure 1b). Here the TMD is represented as a mass, M_1 connected to the primary mass by a spring, K_1 and damper, C_1. With the TMD, the SDOF becomes a two degree of freedom (2DOF) system. As will be shown in the next section, selecting the TMD mass determines how much effective damping can be achieved, while TMD spring rate and damping are used to optimize the TMD performance.

In practice the TMD spring, K_1 may be replaced in part or in whole by a pendulum, while the damper, C_1 is an energy dissipation device. The mass is supported by a pendulum or low-friction bearing system so it can oscillate freely at the [tuned] frequency. When properly tuned, its

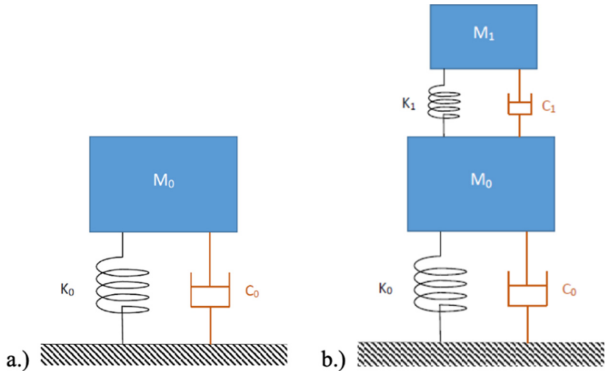


Figure 1. Schematics for: a) SDOF system; and b) SDOF system with TMD.

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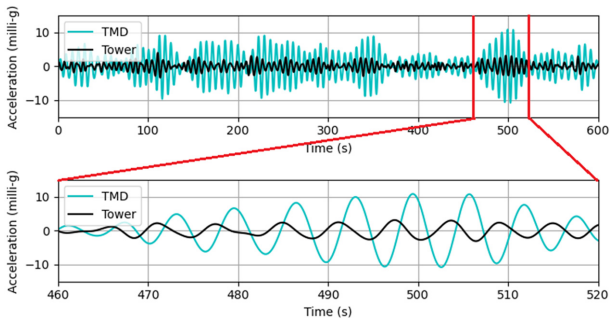


Figure 2. Example tower and TMD acceleration data showing typical phase relation. (Obtained from GERB field measurements)

motion will lag the structure, M_0 by 90 degrees in phase and introduce an inertia force that reduces dynamic amplitudes (Figure 2). Through this reduction of dynamic amplitudes, a TMD effectively increases the damping of the mode of vibration it is added to. The term "effective damping" will be used through the remainder of the paper to describe the total equivalent damping a structure has with a TMD.

TMDs for long periods require pendulums that cannot always be accommodated without loss of valuable real estate. For example, a 0.10 Hz (10 s period) pendulum would already require a 25 m cable when neglecting stiffness contributions from dampers or tuning mechanisms. Hence for longer periods, various techniques may need to be employed to compress the height of the TMD, such as nesting pendulums (e.g. double pendulum described in Table 1a).

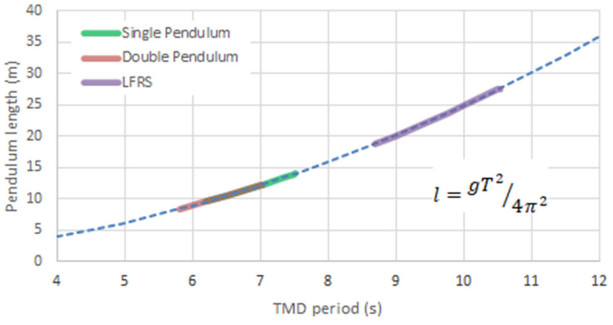


Figure 3. Required pendulum length vs. TMD period for TMDs referenced in Table 1.

Figure 3 shows the ideal pendulum length of an undamped system per TMD period. Three example project TMDs are plotted, employing various TMD styles to accommodate their respective building periods while maintaining dimensions appropriate for the allocated space, primarily the available height. Although the GERB Low Friction Rail System (LFRS™) TMD does not employ a pendulum, it is shown for comparison.

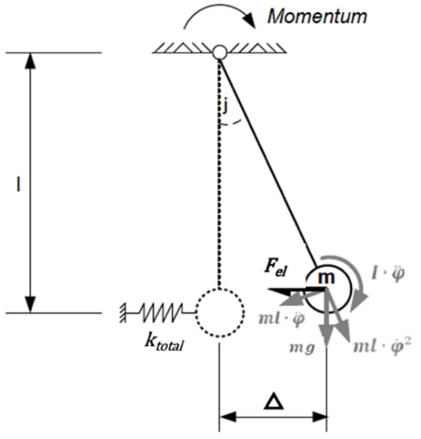
When considering stiffness contributions from the damper and tuning mechanisms (e.g. helical coil tension springs), the system can be represented by the analogous model of Figure 4, where, for small elongations, the equation of motion can be derived from the balance of moments about the hinge. See equation (3).

$$\ddot{\varphi} + \frac{mgl + k_{total} l^2}{I + ml^2} \varphi = 0 \tag{3}$$

Solving the differential equation (3), the natural frequency of the system can be derived as:

$$f = \frac{1}{2\pi} \sqrt{\frac{mgl + k_{total} l^2}{I + ml^2}} \tag{4}$$

The stiffness contributions from these mechanisms will result in a higher TMD natural frequency, which will require compensation with an even longer pendulum. Attempts to compress these dimensions have yielded various TMD arrangements to fit within allotted spaces of slender structures. Table 1 presents three TMD types used in tall buildings over 300 m in height. They exemplify



where:

- gravity load: mg
- tangential inertial force: $ml \cdot \ddot{\varphi}$
- radial centripetal force: $ml \cdot \dot{\varphi}^2$
- moment of inertia: $I \cdot \ddot{\varphi}$
- force due to elastic components: $F_{el} = k_{total} \cdot \Delta$

Figure 4. Analogous Model of pendulum with added stiffness

Table 1a. Example TMD Characteristics

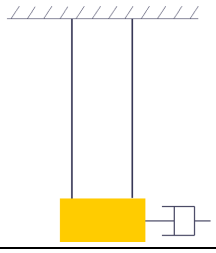
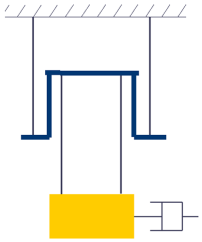
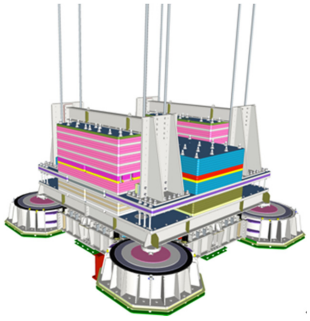
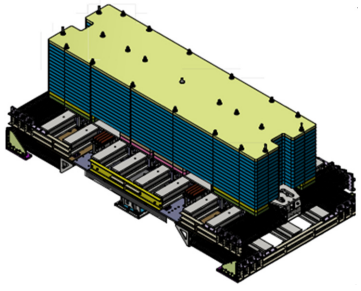
Characteristic	TMD Type		
	Single Pendulum	Double Pendulum	Low Friction Rail System (LFRS™)
Constraints	Long cables for lower frequencies	Additional pendulum frame increases cost and static load	Prefers enclosed space
Degree of structural connections	TMD room ceiling, floor, and potentially exterior walls/columns	TMD room ceiling, floor, and potentially exterior walls/columns	TMD room floor slab
Tuning via	Pendulum Length & Springs	Pendulum Length & Springs	Springs Only
Installed Tuning Freq. Range	Restricted by available pendulum length	Restricted by available pendulum length	No restrictions
Schematic			

Table 1b. Example GERB TMD Implementations

	TMD Type		
	Single Pendulum	Double Pendulum	Low Friction Rail System (LFRS™)
Example Rendering			
Example Photograph (from GERB)			
Moving Mass	590 tonne	405 tonne (+54 t frame)	630 tonne (3 × 210 t)
Design Frequency	0.14 Hz (7.14 sec)	0.15 Hz (6.7 sec)	0.10 Hz (10.0 sec)
TMD height	23 m (75.5 ft)	7.3 m (24 ft)	3 m (10 ft)

successful implementations within crowded mechanical spaces, while accommodating the tuning range around the target primary sway modes.

Table 1a gives general comments regarding their limitations, height advantages and tuning mechanisms.

After validation of the designated building periods (discussed in Section 3), the TMD tuning is accomplished by changing total system stiffness (k_{total}). In a pendulum system, the free length of the pendulum can be adjusted with rope clamps. The rope clamps can fix the TMD pendulum cables at pre-designated elevations to change the active pendulum length, and thus the TMD frequency. For additional fine-tuning, springs may be added or removed. To achieve the highest TMD frequency, the pendulum cable is clamped to establish the shortest possible intended free length and all tuning springs installed. Conversely, all springs would be removed, and the longest pendulum length allowed to achieve the lowest design frequency (longest period). For a GERB Low Friction Rail System (LFRS^{TM}) TMD, stiffness adjustments are only made by springs. This makes this style of TMD the most adaptable to uncertain or changing as-built building properties (elaborated in Section 3).

The TMD damping elements dissipate the kinetic energy of the oscillating mass. Together with proper tuning, the damping elements secure the TMD performance and the resulting increase in effective damping. Non-linear effects and damping stiffness should be considered regarding the tuning of the TMD. The damping elements are essential in controlling TMD motion and can increase the system robustness when mistuning of the TMD occurs. See Section 2.

2. The Importance of Tuning Among Key TMD Parameters

Returning to Figure 1b, the main TMD parameters are mass, M_1 spring stiffness, K_1 and damping, C_1. TMD performance is often presented relative to the mass ratio,

$$\mu = M_1/M_0 \tag{5}$$

and the tuning ratio,

$$\rho = f_1/f_0 = \sqrt{K_1 M_0/K_0 M_1} \tag{6}$$

Generally, the first step in defining the TMD parameters is to determine a target for how much damping needs to be added, and to select a TMD mass ratio that achieves that increase. Figure 5 shows the relationship between effective damping (the damping achieved with a TMD installed) and mass ratio for different levels of inherent structural damping. These effective damping levels assume optimized tuning and optimized TMD damping. Hence, this graph represents the maximum effective damping that can be achieved for a given mass ratio.

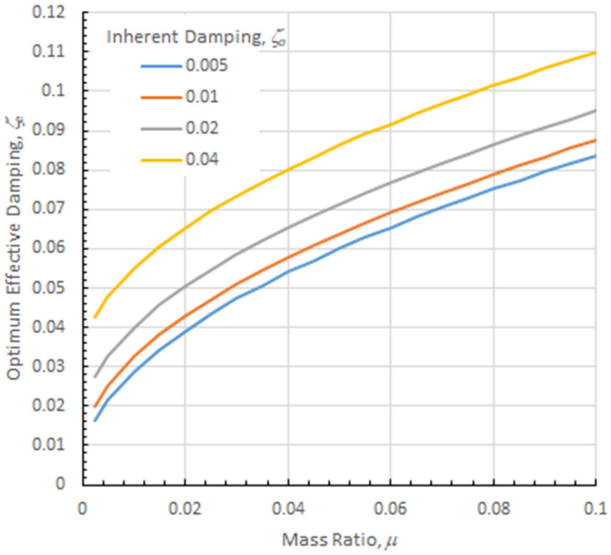


Figure 5. Equivalent damping ratio vs. mass ratio for optimally tuned TMD. (Connor. 2003)

Once the desired mass ratio is selected, the optimum tuning ratio and TMD damping ratio can be calculated. Meinhardt (2008) and den Hartog (1956) provide the following equations for optimum values:

$$\rho_{opt} = \frac{1}{1+\mu} \tag{7}$$

$$\zeta_{opt} = \frac{C_1}{4\pi f_{1,opt} M_1} = \sqrt{\frac{3\mu}{8(1+\mu)^3}} \tag{8}$$

Connor (2003) and Vickery (2006) provide slightly different formulations based on differing assumptions on the type of loading applied to the primary structure, but all provide fairly similar results. As stated, these optimum values allow the maximum level of effective damping to be achieved for a given mass ratio, but it is important to understand how TMD performance is affected when these values are not optimal. Figure 6 graphs effective damping as contours versus TMD damping ratio and tuning ratio with an assumed mass ratio of 2%. From equations (7) and (8), the optimum tuning ratio is $\rho = 98\%$ and optimum TMD damping is $\zeta_{TMD} = 8.5\%$. These optimum values are within the 5% effective damping contour in Figure 6.

Figure 6 shows us two important details about tuning ratio and TMD damping. First is that, while optimization is important to get the highest performance out of a TMD, variations of a few percent from these optimal values do not cause drastic reductions in performance. The second detail is that as effective damping decreases, the widest tuning ratio portion of each contour moves toward higher levels of TMD damping. This provides insight into "robust" TMD design. A robust TMD design is slightly overdamped

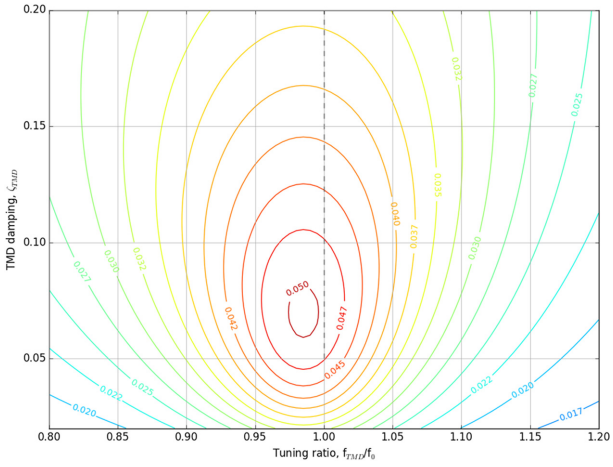


Figure 6. Effective damping contours vs. TMD damping and tuning ratio for $\mu = 2\%$. (Connor. 2003)

so that it provides a minimum level of effective damping over a broad tuning ratio range. Looking back to Figure 6, if a minimum of 4% effective damping is required, a TMD damping ratio of 10% might be chosen. This robust design would provide at least 4% effective damping for tuning ratios from about 92% to 105%.

Robust design could be of particular value when considering a TMD to control strength design loads on a concrete tower. If cracking of the concrete occurs during a major windstorm, thereby changing the tuning ratio, the TMD could still provide the desired effective damping.

3. Checking the Structure During Construction

The design and specification of a TMD for a tall building generally occurs well before the tall building is constructed. Hence, the TMD specifications must be developed based on the dynamic structural properties estimated by calculation or from a structural model. Such estimates rarely predict the as-built natural frequencies perfectly. Figure 7 shows a comparison of measured building periods to predicted building periods from a large number of steel buildings. These results show a substantial amount of scatter and an overall trend that measured periods are typically shorter than predicted.

In addition to uncertainty in the predicted natural frequency, concrete buildings are often examined in both uncracked and "fully-cracked" states. The frequencies predicted with these states can result in differences in frequency in the range of 5% to 15%. Since both states may be possible during the life of a building (uncracked upon completion of construction and fully cracked after a major wind or seismic event), it is important that the TMD can be tuned to, and in between, the frequencies predicted from these two states.

Considering the uncertainty in the predicted natural frequencies and, particularly for concrete structures, the possibility that the natural frequencies may change during

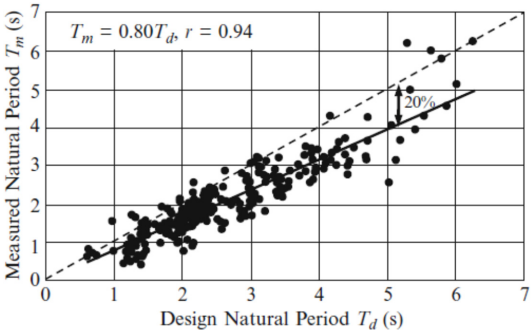


Figure 7. As-built building period vs. predicted period. (Tamura. 2000)

the life of a building, it is practical to specify a tuning range for the TMD. The tuning range establishes the minimum and maximum frequencies to which the TMD can be tuned. From experience, it is common to specify a tuning range of 95% to 115% of the frequency predicted using nominally cracked concrete. Establishing a tuning range for a pendulum-based TMD will control the overall height of the TMD support structure, the TMD rope lengths, and the available locations for rope clamps. The tuning range of an LFRS^{TM} TMD will determine the stiffness and quantity of springs in the design.

Once specified and approved, the fabrication of all TMD components will occur in parallel with the construction of the tower. This allows for optimization of the tower construction schedule by ensuring that all components will be available when TMD installation is to commence, often when the structure is very near or completely "topped out." If at the time of TMD installation it is found that the as-built natural frequency(s) falls outside of the specified tuning range (and fabricated TMD components), there will be schedule and project cost impacts. If the as-built frequency is less than the minimum tuning range frequency, a pendulum-based TMD will likely need longer ropes and a taller support structure. If the as-built frequency is greater than the maximum tuning range frequency, the tuning system of a pendulum-based TMD will need to be modified. For an LFRS^{TM} TMD, if the as-built frequency falls outside of the tuning range, new springs will need to be procured and delivered.

A way to help mitigate schedule and project cost impacts due to inaccurate prediction of as-built natural frequencies is to take measurements of the tower's natural frequencies during construction and use these measurements to check the predicted frequencies of the topped-out structure. One way the measurements can be used is to calibrate a model of the tower in the state of construction at the time of the measurements. Once calibrated to match the measured natural frequencies, the complete tower model is revised with the same calibrations and new completed structure frequencies are predicted.

A simplified method for predicting the final as-built frequencies is to extrapolate based on how much of the

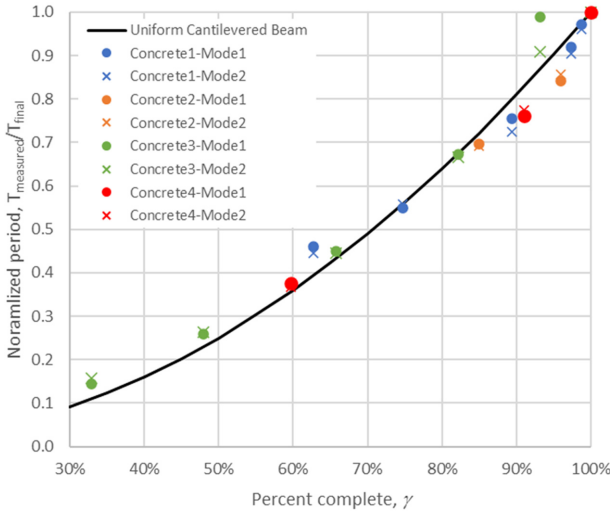


Figure 8. Normalized building period vs. percent complete during construction. (Obtained from GERB field measurements)

tower was constructed when the measurements were taken. Figure 8 plots the normalized building period versus percent complete for the first two sway modes of four different concrete towers recently constructed in New York City. Here the normalized building period is the natural period measured during tower erection divided by the period measured at completion. Percent complete, γ is calculated as the number of completed floors divided by the total number of floors. Figure 8 shows a clear trend when data is reduced in this fashion.

Considering that the four towers used in Figure 8 are all slender and have very little reduction in floor plan area with height, and considering that any reductions in structural stiffness with height are typically accompanied by reductions in floor weight, it is reasonable to approximate these towers as cantilevered uniform beams. The first natural frequency of such a beam will depend on a number of parameters, such as cross-section area and moment of inertia, but most importantly, the frequency is inversely proportional to the square of the beam length. By approximating a building as uniform beam, changes in frequency can be predicted from changes in height alone.

Based on this approximation, the final building period can be estimated using equation (9) and the final building frequency can be estimated with equation (10).

$$T_{final} \cong \frac{1}{\gamma^2} T_{measured} \quad \text{or} \tag{9}$$

$$f_{final} \cong \gamma^2 f_{measured} \tag{10}$$

The accuracy of equations (9) and (10) depends on the assumption that the tower behaves like a cantilevered uniform beam. Some towers may not fit this assumption, such as towers that have significant taper or those with belt walls or outriggers that are not completed at the time of the measurements. For such towers, a more accurate prediction of final tower frequencies will result from structural modeling and calibration, rather than simplified extrapolation.

4. Verification of TMD Tuning

The 2DOF system shown in Figure 1 has two natural frequencies. The undamped natural frequencies can be calculated as the roots of the quadratic equation:

$$\lambda^2 - \left(\frac{k_0+k_1}{m_0} + \frac{k_1}{m_1}\right)\lambda + \frac{k_0 k_1}{m_0 m_1} = 0 \tag{11}$$

Here, λ is the circular frequency, ω squared:

$$\lambda = \omega^2 = 4\pi^2 f^2 \tag{12}$$

where f is a natural frequency. If we know the mass ratio, μ and the tuning ratio, ρ, then we can substitute:

$$m_1 = \mu m_0 \tag{13}$$

$$k_1 = \mu\rho^2 k_0 \tag{14}$$

into quadratic equation (11) and the natural frequencies are found to be:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_0}{m_0}\left[(1+\mu\rho^2+\rho^2) \pm \sqrt{1+2\mu\rho^2-2\rho^2+2\mu\rho^4+\mu^2\rho^4+\frac{\rho^2}{\mu}}\right]} \tag{15}$$

Since the 2DOF system has two distinct natural frequencies, determination of the tuning ratio and TMD tuning from measurements is not straight forward.

There are two ways to determine the TMD frequency. One way is to use a “pluck” test. During a pluck test, the TMD is pulled or displaced and released suddenly. The ensuing TMD motion is measured using an accelerometer. Figure 9 shows an example of simulated TMD acceleration and resulting acceleration response spectrum. In this example, the base structure has a natural frequency of 0.2 Hz and inherent damping of 1%, while the TMD has a mass ratio of 3%, frequency ratio of 98% and internal damping of 9%. The TMD should have a frequency of 0.196 Hz, as shown by the red line in Figure 9. The spectrum (green line) shows peaks near 0.181 Hz and 0.216 Hz as predicted by equation (15). Figure 9 makes it evident that verifying the tuning of a TMD using a pluck test can be cumbersome, and open to potential errors resulting from interpretation of the TMD response.

A second method to determine the TMD frequency is

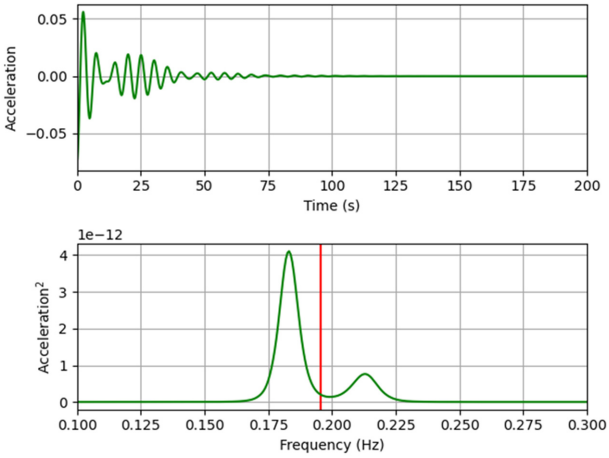


Figure 9. Example time history and response spectrum from TMD pluck test. The red line indicates the TMD frequency.

to use a Frequency Response Function (FRF). The FRF is a transfer function that estimates the response of the TMD as a SDOF system. The FRF magnitude is calculated as:

$$FRF(f) = \sqrt{|P_{11}|/|P_{10}|} \tag{16}$$

where P_{11} is the cross power spectral density from an acceleration measurement on the TMD to the acceleration measurement on the TMD (i.e. the auto-correlation), and P_{10} is the cross power spectral density from an acceleration measurement on the TMD to an acceleration measurement on the tower.

Figure 10 shows example plots of FRFs measured in orthogonal lateral directions. The dotted lines plot the magnitude of the FRF calculated from acceleration measurements. The peak in the FRF occurs at the frequency of the TMD. Curve fitting the measured FRF to the equation for a damped SDOF system may allow for a more reliable measure of the TMD frequency.

Both methods for estimating TMD frequency have advantages and disadvantages. A pluck test requires that the TMD be pulled and quickly released. Such a test requires chain falls, a quick release link and onsite labor to execute and measure during the test. While these tests require specialized equipment and labor, it is beneficial that such testing can be scheduled and conducted at almost any time. Also, as mentioned, the data resulting from a pluck test requires interpretation that may be susceptible to erroneous interpretation.

The FRF method requires that two sensors be used, one on the tower and one on the TMD, and that data from these sensors is sampled simultaneously. The FRF method is most reliable when there is strong signal to noise ratio. In other words, the method works best when the TMD is experiencing high accelerations. This only occurs on very windy days, which is difficult to predict

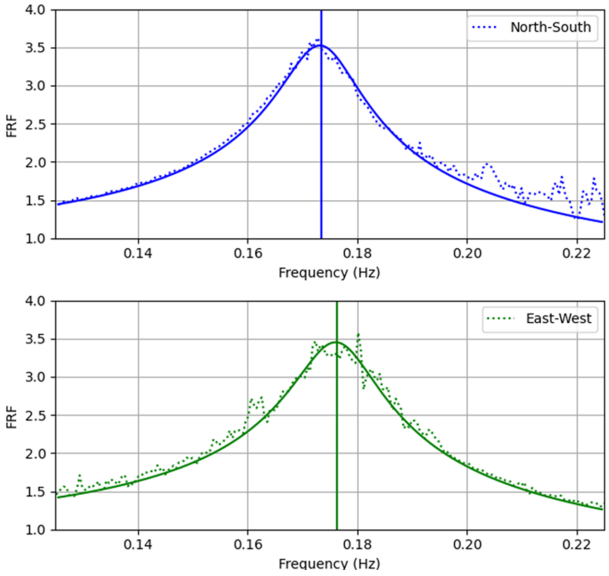


Figure 10. Example FRF plot: dotted lines are calculated from measurements, solid lines are curve fits and estimated frequency from curve fit. (Obtained from GERB field measurements)

with sufficient lead-time. Hence reliance on the FRF method to determine the natural frequency of a TMD may require scheduling data acquisition for a windy day, or continuous data acquisition and reduction of data from a windy day. The FRF method can be easily implemented using data from a permanently installed measurement system. Thus, FRF results can be used as part of a health monitor for the TMD.

5. Conclusions

As more slender towers are being designed and constructed, the implementation of TMDs to control wind-induced motion is becoming more commonplace. This paper presented on the importance of tuning a TMD and procedures to ensure that it is easily verified. While the performance of these TMDs is critical for ensuring occupant comfort, the TMD designer and supplier should also be focused on the successful installation and commissioning of TMDs and take precautions to avoid escalations in project schedule and cost.

High-rise building design teams should expect that an as-built tower's frequencies will not exactly match the predicted frequencies. As presented in Section 3 of this paper, measurements during construction can be used successfully to predict the frequencies of the completed tower. This is an important step in TMD implementation that can help reduce project schedule and cost impacts.

TMD commissioning requires physically tuning the TMD, followed by measurements to confirm the tuning. Ensuring that commissioning is an efficient process is important to staying on budget and on schedule. The GERB LFRS™ TMD was given as an example TMD

implementation. Among other outstanding characteristics, it is perhaps the easiest of all tall-building TMDs to tune. The FRF method was introduced in Section 4 of this paper as means for checking the TMD tuning. This method does not require onsite labor nor specialized equipment for "plucking" the TMD. Hence, the FRF method can be used both during the commissioning process and as part of a long-term TMD monitoring program.

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